# Optimal Experiment Design and Image Reconstruction Using Generative Methods

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## Introduction

**Problem**

Reconstructing images from noisy and indirect measurements is an ill-posed inverse problem critical for many medical and imaging applications, however, obtaining such measurements is expensive time and cost-wise [5].

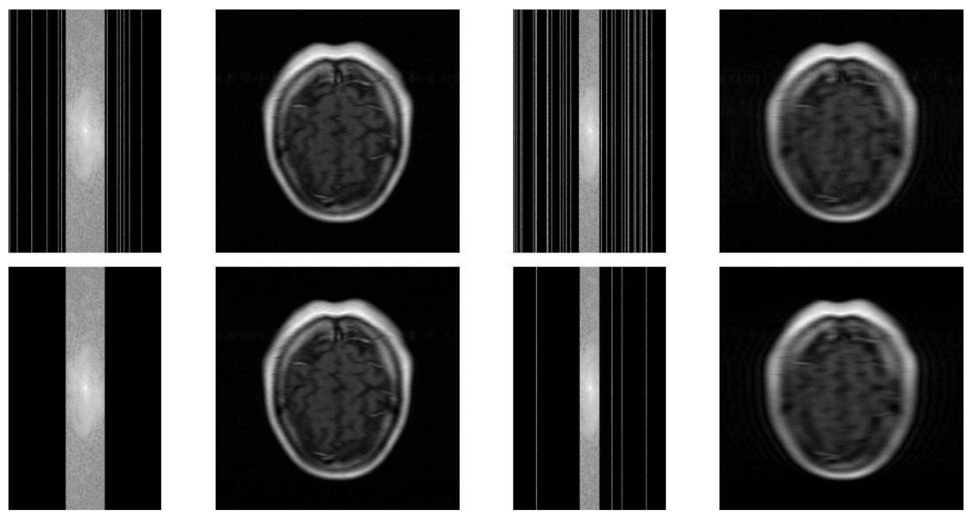


Figure 1: A sample of subsampling masks and reconstructions.

**Our Objective**

- Improve measurement process by finding the best sub-sampled indirect measurement to take, from which quality images are reconstructed.
- Explore how normalizing flows can be leveraged to learn quality image generation by maximizing the effective information gain (EIG) of a jointly learned experimental design [2]. The EIG will be maximized through the minimization of a negative log likelihood objective function.
- Explore introducing a learnable binary mask design parameter $\mathbf{d}$, which represents sub-sampling.

## Our Approach

We propose a model that leverages a Conditional Continuous Normalizing Flow (CCNF) that produces $g_\vartheta^{-1}$ where $x \in X$ is high quality image of the brain and its $y(\mathbf{d})$ condition is an indirect measurement of $x$ that is sub-sampled based on parameter $\mathbf{d}$. Because images are high dimensional inputs, Autoencoders (AE) are also leveraged to reduced inputs before being used to train the CCNF. The overall abstract of the objective or loss we want to minimize is:

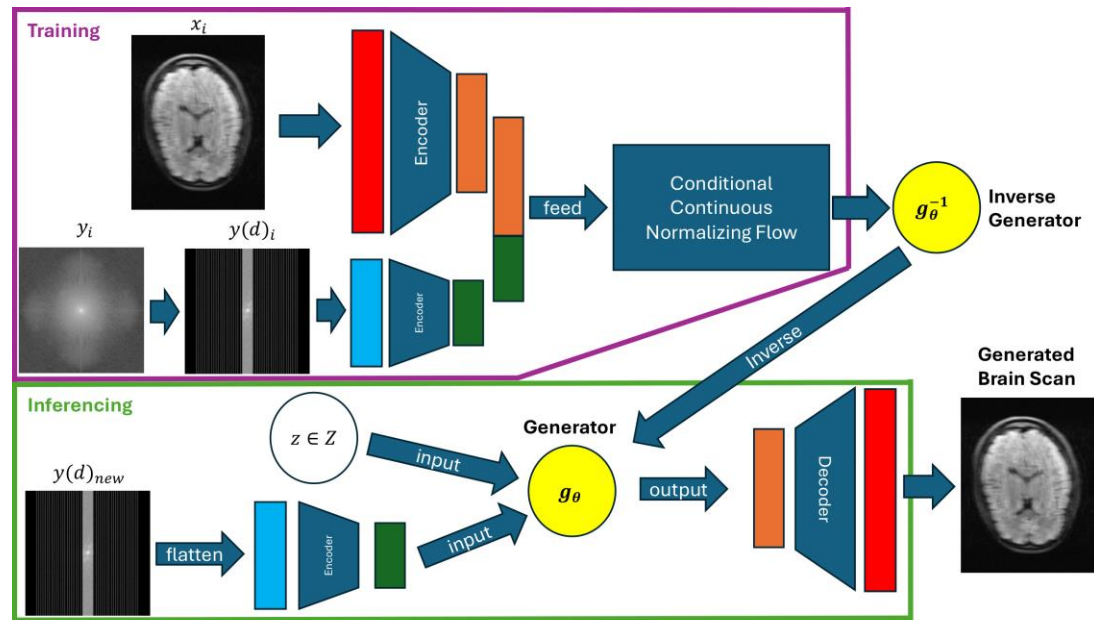
$$\min_{\vartheta,\mathbf{d}} \ell(g_\vartheta^{-1}) + ||\mathbf{d}||_1 \quad (1)$$

where $\vartheta$ represents the learnable weights, $\ell$ represents the negative log-likelihood of the learned distribution $X$ [3], and $||\mathbf{d}||_1$ is the norm of design parameter matrix $\mathbf{d}$.

$$\ell(g_\vartheta^{-1}) = \mathrm{E}[-\log p_\vartheta(x|y,\mathbf{d})] \approx \frac{1}{S}\sum_{i=1}^{S}\left[\frac{1}{2}||g_\vartheta^{-1}(x_i, y_i(\mathbf{d}))||^2 - \log\det\nabla g_\vartheta^{-1}(x_i, y_i(\mathbf{d})) + \frac{n}{2}\log(2\pi)\right] \quad (2)$$

Our motivation for using CCNF is learning the underlying distributions of complex images conditioned on their sub-sampled indirect observations. We aim to promote sparsity of the sub-sampling mask using $\ell_1$ regularization to find how much of the indirect observation can be practically masked.

## Our Model Architecture




## Results

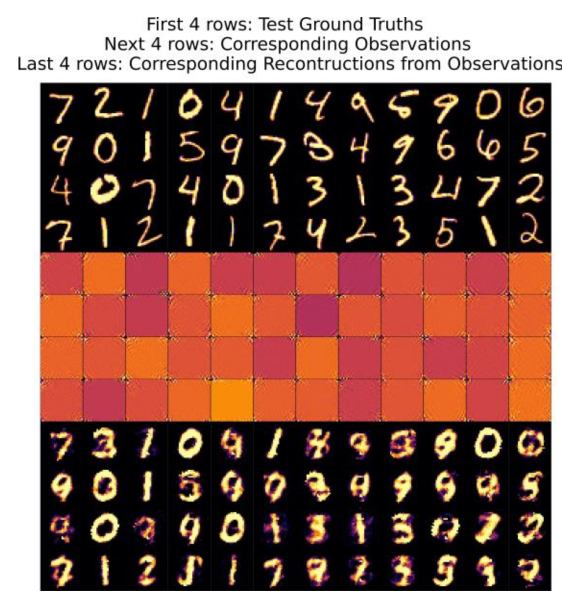



Figure 2: Image reconstructions from our model initialized with a random mask after 30 epochs of training

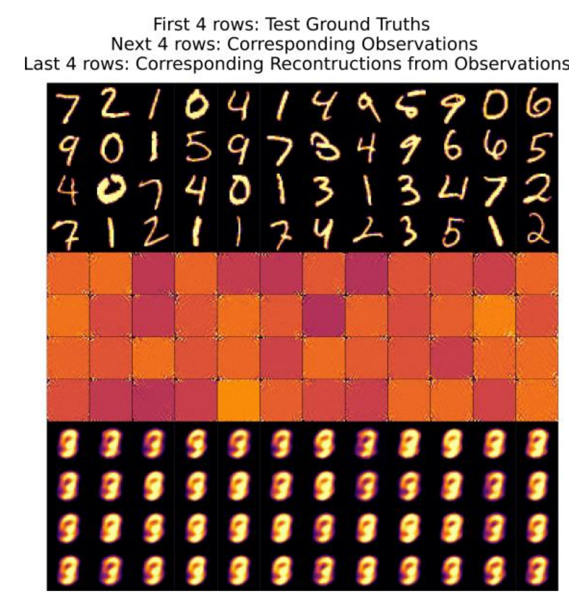



Figure 3: Image reconstructions from our model initialized with a random mask after 100 epochs of training

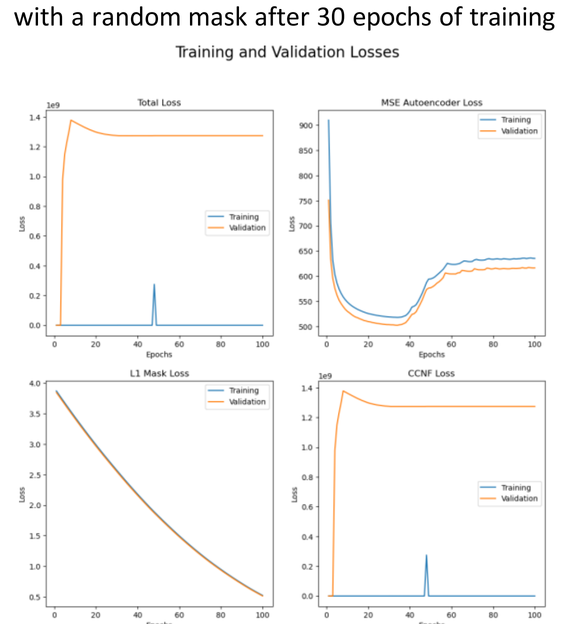



Figure 4: Loss curves from different components of our model initialized with a random mask

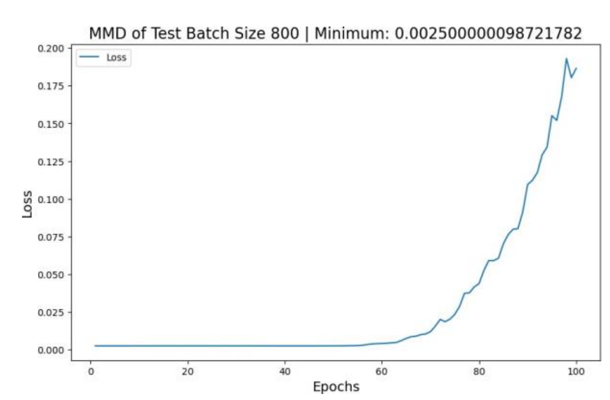



Figure 5: MMD scores throughout epochs of training our model initialized with a random mask

## Results

We compared our generated images to a baseline of images created by the fast iterative shrinking-threshold algorithm (FISTA), which can be used to deblur images [1]. In our case, we used FISTA to deblur the noisy and sub-sampled measurements to reconstruct the quality image. We made these comparisons through maxmium mean discrepency (mmd) calculations using a Gaussian kernel [4]. For each of the methods compared, we measured the MMD between the first test batch of 800 MNIST images and the corresponding 800 image reconstructions.

| Model | FISTA | Our Model (w Random Mask) | Our Model (w Diamond Mask) |
|---|---|---|---|
| MMD | 0.002725518888846134256 | 0.002500000098721782 | 0002500000000040535 |

## Conclusion

- Implemented and tested a version of our proposed model architecture for solving ill posed inverse problems on the MNIST dataset.
- Identified that our image compression stage of the model architecture via autoencoder bottlenecks the performance of our model and negatively impacts the numerical stability of minimizing CCNF loss

**Future Work:**

- Synthesize work with other model architectures that use different conditions, such as FISTA reconstructions
- Further investigate and improve upon several aspects of our model such as our Autoencoder bottleneck
- Conduct more MMD tests on our model and understand the results
- Apply model to MRI and add loss terms to promote the binary mask to be of a practical form for clinical settings

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## References

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